SOLUTION OF HEAT-CONDUCTION EQUATION FOR MATERIALS WITH FADING MEMORY AND HIGH HEAT CONDUCTION

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An analytical solution is obtained of the linearized heat-conduction equation for an isotropic non-deformable body with memory and with high heat conduction.

It was shown in [1] that the solving of the heat-conduction differential equation for materials with variable memory is of great interest. In this article the case of infinitely high heat conduction is studied when the body is spatially isothermic and its temperature is only variable in time. Thus the memory only becomes apparent via the relaxation function of the internal energy.

In the general case the boundary-value problem reduces to the equation [2]

$$h(0)\frac{\partial T(M,\tau)}{\partial\tau}+\int_0^\infty h'(s)\frac{\partial T(M,\tau-s)}{\partial\tau}ds=\lambda(0)\nabla^2T(M,\tau)+\int_0^\infty\lambda'(s)\nabla^2T(M,\tau-s)ds \tag{1}$$

under the initial condition

$$h(0)T(M,0)+\int_0^\infty h'(s)T(M,-s)ds=h(\infty)T_0(M) \tag{2}$$

and the boundary one

$$\lambda(0)\frac{\partial T(N,\tau)}{\partial n}+\int_0^\infty\lambda'(s)\frac{\partial T(N,\tau-s)}{\partial n}ds=\alpha\{T_f(\tau)-T(N,\tau)\}. \tag{3}$$

If one integrates (1)-(3) over the volume V of the body, makes use of the assumption of the infinitely high conductivity $T(N,\tau)=\tilde T(\tau)$, and applies the Ostrogradskii – Gauss formula which transforms a volume integral into a surface one [3], one can obtain the ordinary integrodifferential equation

$$h(0)\frac{d\tilde T(\tau)}{d\tau}+\int_0^\infty h'(s)\frac{d\tilde T(\tau-s)}{d\tau}ds=\frac{\alpha A}{V}\{T_f(\tau)-\tilde T(\tau)\} \tag{4}$$

with the initial condition

$$h(0)\tilde T(0)+\int_0^\infty h'(s)\tilde T(-s)ds=h(\infty)\tilde T_0, \tag{5}$$

where

$$\tilde T(\tau)=\frac{1}{V}\int_V T(M,\tau)dV,\quad \tilde T_0=\frac{1}{V}\int_V T_0(M)dV. \tag{6}$$

By introducing the dimensionless variables

$$\mathrm{Fo}=\frac{\alpha A\tau}{h(\infty)V},\quad \mathrm{Fo}_s=\frac{\alpha As}{h(\infty)V},\quad \mathrm{Fo}_1=\frac{\alpha Ah_1}{h(\infty)V},$$

$$H(\mathrm{Fo}_s)=\frac{h(s)}{h(\infty)},\quad H_0=\frac{h(0)}{h(\infty)},\quad \theta(\mathrm{Fo})=\frac{\tilde T(\tau)-\tilde T_0}{\Delta T},$$

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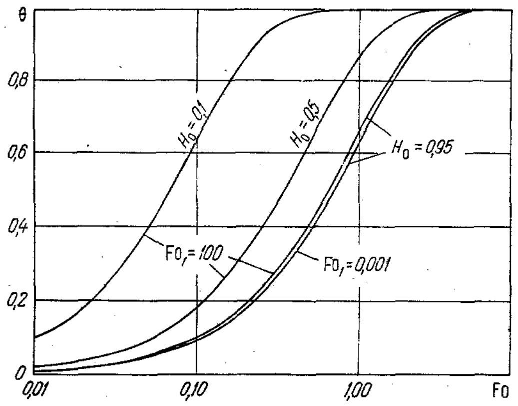

Fig. 1. Dimensionless temperature θ(Fo) versus dimensionless time Fo and the parameters H_0 and Fo_1.

$$\theta_f(\mathrm{Fo}) = \frac{\tilde{T}_f(\tau) - \tilde{T}_0}{\Delta T} \tag{7}$$

one reduces (4) and (5) to

$$H_0 \frac{d\theta(\mathrm{Fo})}{d\mathrm{Fo}} + \int_0^\infty H'(\mathrm{Fo}_s) \frac{d\theta(\mathrm{Fo} - \mathrm{Fo}_s)}{d\mathrm{Fo}} d\mathrm{Fo}_s + \theta(\mathrm{Fo}) = \theta_p(\mathrm{Fo}), \tag{8}$$

$$H_0\theta(0) = \int_0^\infty H'(\mathrm{Fo}_s)\,\theta(-\mathrm{Fo}_s)\,d\mathrm{Fo}_s = 0. \tag{9}$$

If one applies the Laplace transformation one obtains the following solution for the transform:

$$\bar{\theta}(p) = \frac{\bar{\theta}_f(p)}{p^2\bar{H}(p) + 1}. \tag{10}$$

It is now assumed that the temperature of the outer medium is constant $\theta_f(\mathrm{Fo}) = 1$. To obtain the original of the temperature it is necessary to choose a more specific form for the relaxation function h(s) of the internal energy. In view of the constraining conditions formulated in [2] this function can be written in the following dimensionless form:

$$H(\mathrm{Fo}_s) = 1 - (1 - H_0)\exp[-\mathrm{Fo}/\mathrm{Fo}_s]. \tag{11}$$

By employing (11) the expression (10) can be transformed as follows:

$$\theta(p) = \frac{1}{p} \frac{p = \dfrac{1}{\mathrm{Fo}_1}}{H_0p^2 + \left(1 + \dfrac{1}{\mathrm{Fo}_1}\right)p + \dfrac{1}{\mathrm{Fo}_1}}. \tag{12}$$

The inverse of (12) can easily be found in the tables [4] and is of the form

$$\theta(\mathrm{Fo}) = 1 - \sum_{j=1}^{2} \frac{1 + \gamma_j}{1 - H_0 p_j \gamma_j} \exp[p_j \mathrm{Fo}], \tag{13}$$

where

$$\gamma_j = \mathrm{Fo}_1 p_j = -\frac{1 + \mathrm{Fo}_1}{2H_0}\left\{1 + (-1)^j \sqrt{1 - \frac{4H_0\mathrm{Fo}_1}{(1 + \mathrm{Fo}_1)^2}}\right\}. \tag{14}$$

If $\mathrm{Fo}_1 \to 0$, then in calculating p_1 one obtains the indeterminate form 0/0. For this case (13) has been transformed into

$$\theta(\mathrm{Fo}) = 1 - \frac{\exp[-\mathrm{Fo}/(1 + \mathrm{Fo}_1)]}{1 + \mathrm{Fo}_1\left(1 - \dfrac{H_0}{1 + \mathrm{Fo}_1}\right)}. \tag{15}$$

To illustrate the above the dimensionless temperature θ is shown against the dimensionless time Fo with parameters, namely, the value of the relaxation function H_0 of the internal energy at a current instant, and the dimensionless relaxation time Fo_1. In the classical heat-conduction theory one has $H_0 = 1$ and $Fo_1 = 0$. Therefore, as seen from Fig. 1, when H_0 is suitably high, say, $H_0 = 0.95$, a change in Fo_1 has a slight effect on the results. Similarly, for $Fo_1 \to 0$ a variation in the parameter H_0 has no noticeable effect on the nonstationary temperatures. However, for small values of H_0 and large Fo_1 there is a considerable effect of the fading memory. This is clear from the diagram for $H_0 = 0.1$ and $Fo_1 = 100$.

NOTATION

T, temperature; T_0, equilibrium temperature; T_f, temperature of the surrounding medium; M, point of volume; N, point of surface; s, integration variable; p, Laplace variable; h(s), relaxation function of internal energy; $\lambda(s)$, relaxation function of heat flow; α, heat-exchange coefficient; τ, time; Fo, dimensionless time; Fo_s, dimensionless integration variable; H_0, the value of dimensionless relaxation function of internal energy at current time; H(s), dimensionless relaxation function of internal energy; θ, dimensionless temperature; Fo_1, dimensionless relaxation time of internal energy; θ_f, dimensionless temperature of the medium.

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NONSTATIONARY FILTRATION OF A THREE-PHASE MIXTURE TAKING ACCOUNT OF GRAVITATION

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A method of solution and results of calculations are presented for the problem of displacement of gasified petroleum by water in an inclined stratum.

The process of displacement of gasified petroleum by water in an inclined stratum which is assumed homogeneous is examined in this paper; the physical properties of the fluids and collector are considered known. It is also kept in mind that the process is isothermal and that thermodynamic equilibrium is built up instantaneously between coexisting phases. We neglect the influence of capillary forces.

Under the assumptions mentioned, the process of one-dimensional nonstationary filtration of a three-phase mixture (water — petroleum — gas) is described by a nonlinear system of second-order partial differential equations (see [1]), which is written as follows in dimensionless form:

$$\frac{\partial}{\partial x}\left[\frac{k_p s_p}{\mu_p \beta_p}\left(\frac{\partial p}{\partial x}+a_p\right)+\frac{k_g \mu_2}{\mu_g \beta_g}\left(\frac{\partial p}{\partial x}+a_g \gamma_g\right)\right]=\frac{\partial}{\partial t}\left[\frac{(1-\sigma_g-\sigma_w) s_p}{\beta_p}+\frac{\sigma_g}{\beta_g}\right],$$

$$\frac{\partial}{\partial x}\left[\frac{k_p}{\mu_p \beta_p}\left(\frac{\partial p}{\partial x}+a_p\right)\right]=\frac{\partial}{\partial t}\left(\frac{1-\sigma_g-\sigma_w}{\beta_p}\right), \qquad (1)$$

$$\frac{\partial}{\partial x}\left[\frac{k_w \mu_1}{\mu_w \beta_w}\left(\frac{\partial p}{\partial x}+a_w\right)\right]=\frac{\partial}{\partial t}\left(\frac{\sigma_w}{\beta_w}\right).$$

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